SOLVING IT *LIKE A MATHEMATICIAN*

Problem solving

Problem solving is a valuable skill. Being able to calmly and logically analyse some information and process this to solve a problem is useful in mathematics, of course, but also in most other subjects, careers and many areas of everyday life. Your computer is broken? You need to plan a journey? You need to cook several dishes to all be ready at the same time? Then, you need to process what information you have and use this to move to a solution. You are problem solving.

Sometimes you can look at a problem and instantly know how to solve it. This is good, but you aren't really learning a problem solving process. It is important that you sometimes try problems or puzzles that you don't find easy because at some point you will start to encounter hard problems and before then you need to practice your approach to problem solving. Below is some advice on three stages of problem solving. Each stage is equally important. On the reverse are some problems you can try: some you may find easy and others you may not be able to solve yet.

1. Plan

Many people expect to read the problem and immediately attempt a solution. This is rarely possible, and when their first attempt fails they may give up. You can avoid this trap by taking the time to understand the problem and plan your approach.

Stop and really read the problem.
Ask yourself: What am I being asked to do? What information have I been given? What information is missing? What would a solution look like?
Draw a picture. Introduce suitable notation for the information you have been given and the information you are being asked for.
Have you seen a similar problem? How did you solve that one?
Separate the problem into smaller parts and examine them separately.
Can you write the problem in a different way? Try it. Is what you have written actually the same problem? If not, what is different? If it is, can you solve this version of the problem?
Can you solve a related problem? Can you remove part of the restriction and solve a more general problem? Or, can you come up with a specific example and solve that? Do your solutions help you plan to approach the main problem?

2. Carry out your plan

When your plan is ready, put it in action

Carry out your plan. Check each step. Is each step correct?
You should expect to be stuck quite a lot of the time. Recognise that you are stuck and accept it. Calmly review where you are and try to get unstuck. It is okay to wait and mull over the problem for a while.
If you are sure your plan cannot work, you may need to return to the Plan stage.

3. Review

If you find a solution or are about to give up, move to the review stage.
It is important to check your solution is correct and think about what you have learned about this type of problem and about the problem solving process.

First, if you found a solution, check it is correct.
Can you check your solution is correct? Does it answer the original problem? Can you get the same solution from a different method? Can you work from your solution to get back to the original problem and, doing so, is the problem you get to the same as the original problem you tried to solve?

Second, reflect on what happened.
Remember that the point of solving problems is not just to get marks in tests and exams. The purpose here is to think about what has happened and see what you can learn.
Can you use the method, or the result, for some other problem? Can you write down a new problem that you can now solve? Is the problem you have solved part of a wider family of problems? Can your method be adapted to solve more of them?
Think about the process you took and particularly any dead ends you went down. What went wrong? Could you have avoided the dead ends, or were they a necessary part of solving the problem? What do you wish you had known when you first attempted the problem?

The advice here was written by Peter Rowlett and is heavily inspired by Pólya's *How to Solve It* and Mason, Burton and Stacey's *Thinking Mathematically*.
You can find more puzzles like the ones given on the reverse in puzzle books by authors such as Henry Dudeney, Sam Loyd, Martin Gardner and Ian Stewart.
You can find this advice sheet and more puzzles at www.peterrowlett.net/solveit

Some puzzles

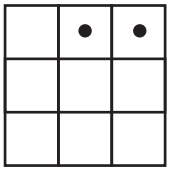
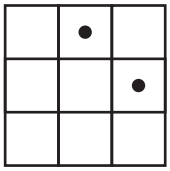
Don't feel you have to try these in any particular order, or even that you have to try them all. Pick one that you like the look of and start there.

1.

Twelve friends meet and shake hands. How many hand shakes were there?
What if there were 100 friends?

2.

How many ways are there of drawing two adjacent dots on a 3x3 grid?

e.g.  

3.

Two market sellers are selling apples. The first sells three apples for 1p; the second sells two apples for 1p. Both are called away on an emergency with 30 apples each left to sell. They ask a friend to sell these at five for 2p.
How much money would they have made selling them individually? How much does their friend make selling them?
Why is there a difference?

4.

There is a boy and a girl.
One of them, with black hair, says "I'm a boy".
One of them, with red hair says, "I'm a girl".
At least one of them is lying. What colour is the boy's hair?

5.

Why are manhole covers not square?

6.

Use three equal digits to make 24. The solution for 8 is obvious: 8+8+8.
For 2 we require a little trickery: 22+2.
Using addition, subtraction, multiplication, division, powers, square roots and factorials, find solutions using three digits 3, 4, 5 and 9.

7.

You have three jugs - one holds 8 litres, the second holds 5 litres and the third holds 3 litres.
The 8 litre jug is full of water. You need 4 litres of water. How can you pour liquid between the jugs to end up with exactly 4 litres?

8.

What is the next number in this sequence?

1, 2, 4, 7, 8, 11, 14, 16, 17, 19, 22, 26, 28, 29, 41, 44

Clue: write down what is not there.

9.

If you cut a donut with three straight cuts, what is the largest number of pieces you can make?