

Fifteen puzzle

How to play:

A valid move means moving one token that is adjacent to the empty space into the empty space.

Setup the tokens in the starting position given to the right. Without looking, ask a friend to move the pieces – only using valid moves – until the puzzle is scrambled. Then you must unscramble it by making valid moves to get the puzzle back into order.

Starting position:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

History:

This puzzle was invented by Noyes Palmer Chapman, a postmaster from a village in New York State, as early as 1874 and created a craze in America and Europe in 1880. Famous puzzler Sam Loyd claimed later to have invented the puzzle but this isn't true. Loyd released a version of the game with the arrangement on the right – called the 14-15 puzzle – and offered a \$1,000 reward for a correct solution. Unfortunately this arrangement cannot be solved! Can you see why?

Loyd's 14-15 puzzle:

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

Advanced

How to decide if a solution is possible:

For each cell, moving left to right and top to bottom, write down how many cells contain a number that is lower than the number in the current cell. For example, in the grid on the right, the first cell contains 13 and there are 12 cells still to be counted that contain a lower number. The next cell contains 10 and there are 9 cells still to be counted that contain a lower number. The next contains 11 and there are 9 cells to be counted that contain a lower number – we don't include 10 because 10 has already been counted. Continue and get: 12, 9, 9, 5, 4, 4, 3, 3, 0, 3, 3, 2, 1, 1, and 0.

13	10	11	6
5	7	4	8
1	12	14	9
3	15	2	

Now add these numbers together. In our example, this gives 59. Now add the number of the row on which the blank square appears, in this case the blank space is on the 4th row so this number is 4. Adding 4 to 59 gives 63. If this number is even, a solution is possible, otherwise it is not. Since 63 is odd, the example given here cannot be solved.

In the case of Sam Loyd's 14-15 puzzle, the values of the 15 cells are: 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0. The blank space is on row 4. Then the sum is 5 and the puzzle is impossible.

Try for yourself: Place the tokens randomly and calculate whether a solution is possible before attempting to solve it.

You can find this puzzle sheet and more puzzles at www.peterrowlett.net/solveit